

CHAPTER 4

SOLUTION OF BASIC EQUATIONS OF ELECTRODYNAMICS

From the intrinsic evidence of his creation, the Great Architect of the Universe now begins to appear as a pure mathematician.

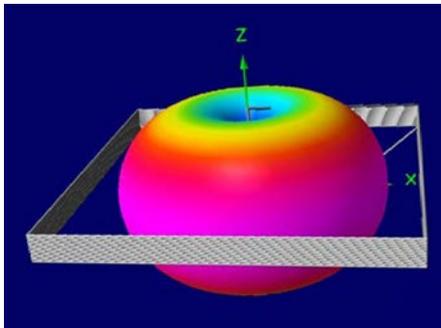
Sir James Jeans

British physicist, astronomer, and mathematician

You can learn in Chapter 4

The EM energy conservation law established in Chapter 3 clearly predicts the existence of EM waves and their ability to move energy in space. The most remarkable fact that such energy transportation is “self-organized and self-sustained” as long as the *instantaneous* electric fields are converted into *instantaneous* magnetic fields and vice versa. If so, the energy exchange between these two fields as two different manifestations of the same EM phenomenon can continue indefinitely long as soon as such transfer is free of loss. However, this observation is not enough if our goal as engineers is to develop very sophisticated modern power or communication systems. For example, we need to know how to manage transportation EM energy in given direction with minimum loss, how EM waves interact with material objects surrounding us, what is the most effective ways of EM waves generation, and hundreds and hundreds of more different and often much more complicated questions. The primary goal of this chapter is to give our readers the sense of confidence to live and operate in the world of electrodynamics. We tried to play down the mathematical transformations as much as possible and present particular final expressions in the purest possible form. We hope that the multiple comments and drawings might help. Readers who already know this material are encouraged to skim through the chapter for notation.

After all, we’re not interested in Maxwell’s equations for their own sake but wish to understand what equations tell us about the physical world. To do so, we’ve condensed two Maxwell’s curl equations down into a single differential equation of second order involving nothing but E or H getting thereby Helmholtz’s or wave equations. You learn how to simplify the solution of these



equations introducing the vector and scalar potentials with a clear physical interpretation through the unit dimension analysis. In free space, the most straightforward solution of these wave equations are waves traveling at a speed of light, c .

You can find in this chapter wide-ranging information how to manage three main type of elementary radiators: electrical, magnetic, and Huygens. Each of them is a small construction brick you may find practically in any antenna designs from the past, up-to-date and future. You will become familiar how to do it in Chapter 5. The 3D radiation pattern of small rectangular loop antenna as an example of the magnetic radiator is depicted here.

The last topic in this chapter is the effect skin-depth in the highly conductive material. To make the material more close to engineering practice, we’ve considered the impact of metal surface roughness on attenuation of EM waves and introduced the concept of surface conductivity and resistivity.

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