

CHAPTER 3

POYNTING'S THEOREM



JH Poynting

*The whole story of ... creation can be told with perfect accuracy and completeness in the six words:
God said, Let there be light.*

Sir James Jeans

British physicist, astronomer, and mathematician

You can learn in Chapter 3

In Section 1.2 of Chapter 1, we already discussed the ultimate relationship between symmetries of nature and conservation laws, most fundamental principles of physics, and stated that Maxwell's equations possess all the required symmetries and, therefore, can be derived from conservation laws. We preferred not follow this “ultra-triathlon” mathematical path and took “stress-free” axiomatic way to “get” Maxwell's equations. Evidently, if all in all we have done in Chapter 1 is correct, the EM field energy and power conservation laws must be not only the part of Maxwell's equations but must be and will be derivable from them. It will not be an exaggeration to say that Poynting's theorem proving the conservation of energy for EM fields is the electrodynamics groundwork. By today, any result *in macroscopic electrodynamics* that is in contrary to this theorem should be treated as highly doubtful or an error.

Warning. Don't expect to grasp the material in this chapter wholly on the first pass. Please read it one more time sitting with a pad of paper, reviewing the main ideas and writing in words what the equations say in symbols. Trust us that onion is worth peeling because Poynting's theorem is universal and very powerful tool to cross-check any simulation, analytical or especial numerical, while checking its numerical convergence.

You learn how to use the intimate relationship between the EM stored or dissipated energy, and the lumped circuitry element parameters that leads to a much better understanding of circuit theory as well as Maxwell's equations. Poynting's theorem tells you that any physical element or any part of it where the stored electrical energy much exceeds the magnetic one behaves mostly as a capacitor. The element with a contrariwise combination of energy acts primarily as an inductance and so on. Moreover, from Maxwell's equations follow that the instantaneous electric fields are the source of instantaneous magnetic fields and vs. making electric and magnetic fields are almost inseparable. Then you may take fresh look at the basic concepts of circuit theory such as voltage, current, inductor, capacitor, or resistor and recognize that, for example, in the case of time-varying fields you should include into equivalent circuit of a capacitor small but thereby nonzero “parasitic” inductor and resistor, etc.

Poynting's theorem is the tool that make you capable to construct the multiple equivalent circuits of any device of any complexity at any frequencies including optical. Afterward, you might even forget about Maxwell's equations and follow an accustomed way of analysis or synthesis with support of modern circuit theory. Furthermore, you will understand why the same RF element might have diverse equivalent circuits at different frequencies making RF circuitry design a quite challenging task but opening path to new kind of RF devices like resonators.

Ultimately, Maxwell's equations describe in the sense of macroscopic electrodynamics all possible varieties of EM fields that can be generated by the probably enormous diversity of sources surrounded by an infinite mixture of physical objects with multiple combinations of dielectric and magnetic parameters. Poynting's theorem and following from it the Uniqueness Theorem teach you how any real EM problem should be adequately conveyed to provide one and only one solution. We will help you understand why such additional settings as initial, boundary, radiation, and edge conditions should state as long as you care to get the experimentally verified solutions.

With Poynting's theorem backing we introduce you to the concept of measurement-friendly reflection coefficient to bypass the difficulties related to an ambiguous definition of such custom quantities as voltage, current, and impedance at high frequencies. As a result, you will become on close terms with Smith chart, one of the most useful graphical tools for high frequency circuit



analysis and synthesis, and Foster's Reactance Theorem, another useful tool to check the results of analytical or computer simulations.

The last paragraphs give you idea how using Poynting's theorem to establish Lorentz's reciprocity theorem, critical issue in antenna (impressive anechoic antenna test chamber of the Ohio State University is shown here) and microwave theory, measurement, and development.

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