Design and Analysis of Microstrip Patch Antenna Arrays

Ahmed Fatthi Alsager
Design and Analysis of Microstrip Patch Antenna Arrays

Ahmed Fatthi Alsager, ahmed4912@yahoo.com

Master thesis

Subject Category: Electrical Engineering– Communication and Signal processing

University College of Borås
School of Engineering
SE-501 90 BORÅS
Telephone +46 033 435 4640

Examiner: Samir Al-mulla, Samir.al-mulla@hb.se
Supervisor: Samir Al-mulla
Supervisor, address: University College of Borås
SE-501 90 BORÅS

Date: 2011 January
Keywords: Antenna, Microstrip Antenna, Array
To

My Parents
ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to the School of Engineering in the University of Borås for the effective contribution in carrying out this thesis.

My deepest appreciation is due to my teacher and supervisor Dr. Samir Al-Mulla.

I would like also to thank Mr. Tomas Södergren for the assistance and support he offered to me.

I would like to mention the significant help I have got from:

   Holders Technology
   Cogra Pro AB
   Technical Research Institute of Sweden SP

I am very grateful to them for supplying the materials, manufacturing the antennas, and testing them.

My heartiest thanks and deepest appreciation is due to my parents, my wife, and my brothers and sisters for standing beside me, encouraging and supporting me all the time I have been working on this thesis.

Thanks to all those who assisted me in all terms and helped me to bring out this work.

Ahmed Fattih Alsager
ABSTRACT

The performance and advantages of microstrip patch antennas such as low weight, low profile, and low cost made them the perfect choice for communication systems engineers. They have the capability to integrate with microwave circuits and therefore they are very well suited for applications such as cell devices, WLAN applications, navigation systems and many others.

In this thesis; a compact rectangular patch antennas are designed and tested for GPS devices at 1.57542 GHz, and for a satellite TV signal at 11.843 GHz and 11.919 GHz. The final part of this work has been concentrated on studying an array antenna with two and four elements. The antennas of the design examples of this work has been manufactured and tested in laboratory.
Table of contents

Acknowledgments 4
Abstract 5

1 Chapter One: Introduction to Antennas 8
   1.1. Introduction
   1.2. Simple Dipole Antenna
   1.3. Radiation Pattern
   1.4. Directional Antennas
   1.5. Microwave Antennas
      1. Parabolic Reflector
      2. Horn Antenna

2 Chapter Two: Microstrip Antenna 16
   2.1. Introduction
   2.2. Types of Patch Antennas
   2.3. Feeding Methods
      2.3.1 Microstrip Line Feed
      2.3.2 Coaxial Feed (Coplanar Feed)
      2.3.3 Proximity Coupling
      2.3.4 Aperture Coupling

3 Chapter Three: Methods of Analysis 22
   3.1 Transmission Line Method
   3.2 Cavity Method
   3.3 The Ground Plane

4 Chapter Four: Antenna Parameters 30
   4.1 Radiation Pattern
   4.2 Efficiency and Quality Factor
   4.3 Directivity and Gain
   4.4 Impedance Matching
   4.5 Return Loss
   4.6 Polarization

5 Chapter Five: Design Method 39
   5.1 Design Methods
5.2 Example one: GPS antenna
   Scattering parameters

5.3 Example Two: Satellite TV Signal

6  Chapter Six: Array Antenna

   6.1 Introduction
   6.2 Two Element Array
   6.3 Linear Array
   6.4 Example 3: Microstrip Array Antenna

Appendix A: 77

Appendix B: 78

References 79
1. Chapter One: Introduction to Antennas

1.1. Introduction

Antennas are key components of any wireless system [1, and 10]. An antenna is a device that transmits and/or receives electromagnetic waves. Most antennas are resonant devices, which operate efficiently over a relatively narrow frequency band. An antenna must be tuned to the same frequency band that the radio system to which it is connected operates in, otherwise reception and/or transmission will be impaired. The receiving antenna as a part in the system is responsible of turning the electromagnetic waves into its original form (electrical signal in wire). The properties of the transmitting and receiving antennas are fully represented by Maxwell’s equations (equations (4.31) to (4.34)). The dipole antenna was the first type of antenna to be ever used and the simplest one to study and understand, it is a straight wire fed from the centre. To tune the wire to be effective to transmit and receive electromagnetic waves, the length of it should be half the wavelength of the operating frequency. For example for a frequency of 300MHz with a wavelength 1 metre, 0.5 metre antenna is required.

\[ \mathbf{J} = \mathbf{n} \times \mathbf{H} \]  \hspace{1cm} (1.1)

Figure (1.1) Equivalent electric and magnetic surface current densities of a dipole antenna

1.2. Simple Dipole Antenna

To understand the function of the antenna we consider the antenna to be enclosed by a surface volume and radiate in homogenous space (free space). It can be replaced by an electromagnetic source of two current densities (figure (1.1)):

If \( \mathbf{J} \) is the Electric current density

\[ J = \mathbf{n} \times H \]
And magnetic current density $M$

$$M = -\vec{n} \times E$$  \hspace{1cm} (1.2)

Where

- $H = \text{Magnetic field intensity (A/m)}$.
- $E = \text{Electric field density (V/m)}$.

![Figure (1.2) Spherical coordinates for an electric dipole](image)

$$E_{\theta} = jk_{o} \sqrt{\frac{\mu_{o}}{\varepsilon_{o}}} \frac{ldz}{4\pi} \sin\theta \frac{e^{-jk_{o}r}}{r} = \sqrt{\frac{\mu_{o}}{\varepsilon_{o}}} H_{\phi}$$  \hspace{1cm} (1.3)

$$H_{\phi} = jk_{o} \frac{ldz}{4\pi} \sin\theta \frac{e^{-jk_{o}r}}{r} = \sqrt{\frac{\varepsilon_{o}}{\mu_{o}}} E_{\theta}$$  \hspace{1cm} (1.4)

Where:

- $Z_{o} = \sqrt{\frac{\varepsilon_{o}}{\mu_{o}}}$ is free space impedance.

- $\varepsilon_{o}$ = permittivity of free space.
\[ \mu_0 = \text{permeability of free space.} \]

\[ r = \text{distance to observation point (in metre).} \]

The complex pointing vector \( S \) in the far field is given by:

\[ S = E \times H^* = k_0^2 \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{ldz}{16\pi^2} \frac{\sin^2\theta}{r^2} \hat{r} \]  \hspace{1cm} (1.5)

Equations (1.3) (1.4) (1.5) are well known in communication engineering and can be seen in almost all antenna books, we will not consume time in prove them now, we only mention them, further study of these equations can be found in [1, 3, 4, 5, 6, 8, and 10].

### 1.3. Radiation Pattern

The radiation pattern of an antenna is an electromagnetic wave, and we are interesting in calculating and measuring the strength of this electromagnetic wave at a distant point. This distant point is somewhere in the space where the wave is considered to be plane wave and normal to the direction of the antenna.

Radiation pattern is the variation of the electric field as a function of angle and has two field components: an \( E \) field vector and an \( H \) field vector as shown in figure (1.2).

The radiation pattern can be represented in either Cartesian or polar coordinates, as shown in figure (1.3).

We will discuss later in this work more parameters of the antenna in connection with microstrip patch antenna like radiation pattern, efficiency, quality factor, directivity, gain, and more.

![Figure (1.3) Radiation pattern; cartesian and polar diagram](image)
1.4. Directional Antennas

Speaking of dipole antenna is speaking of omnidirectional antennas which radiates in all directions. Directed antennas are another category of antennas. The term directional antenna is used for antennas which radiates power in focused and specific direction. Directional antennas can be fixed in a specific location and directed towards the receiver (or transmitter) such as in microwave communications, or it can require rotation facilities as in radars.

The ability of the antenna in focusing power in one direction more than other directions is a measure of quality of the antenna and it is often expressed by the terms gain, directivity, front to back ratio, half-power beam width HPBW, and many other factors and parameters of the antenna. These parameters will be discussed later. In general we can define the front to back ratio (or Half-Power beam width HPBW) as:

\[
20 \log \frac{\text{far field strength in main direction of radiation}}{\text{far field strength in backwards direction}}
\]

Another factor is important in studying antennas is the angle between two directions of the radiation pattern where the field strength is reduced to 70% of its total value (-3dB) and the radiated power to 50% of its value; figure (1.4):

![Radiation pattern of a typical directional antenna](image)

**Figure (1.4)** Radiation pattern of a typical directional antenna

We can get a good understanding of the directional antenna by setting two omnidirectional antennas with \(\lambda/4\) distance between them, as shown in figure (1.5):
Let’s feed the two antennas with signals of different phase of $90^\circ$. The radiation pattern of this combination is shown in figure (1.6).

When combining two or more elements to build an antenna with a specific defined distance and specific defined phase shifters, we can obtain the desired radiation pattern. A very common type of antennas is Yagi-Uda antenna uses this principle. It can be seen in most households to receive UHF and VHF TV signals. Yagi-Uda antenna uses the principle of radiation coupling, in which the feeding is to one element and other elements will be activated by it.

1.5. Microwave Antennas

1. Parabolic Reflector

The parabolic reflector dish is the most common type of antennas when high gain is required. It has been used since the early of 1900s. It became very popular under the World War II in radar applications and in the present time it can be seen in almost household. The main advantage of the parabolic antenna is the large gain and directivity; however the main disadvantage is the big size dishes which are not easy to mount and have large windage.

The principle is the same as the optical mirror reflector, when the source of beams is at the focus on the axis of the parabola, the reflected beam from the parabola will be parallel to the axis of the parabola; figure (1.7)
The beam width of the parabolic reflector antenna is related to the wavelength [13].

\[ \theta = k \frac{\lambda}{D} \]  \hspace{1cm} (1.6)

Where \( \theta \) is the beamwidth.

D is the diameter of the reflector.

\( \lambda \) is the wavelength.

K is a constant depending on the illumination from the primary feed with a value of about 60.

The gain of the parabolic reflector can be obtained from:

\[ G = \frac{(\pi D)^2}{\lambda^2} \eta \]  \hspace{1cm} (1.7)

Where \( \eta \) is the efficiency.

From equation (1.6) we can note that the beam width of the antenna increases as the diameter of parabolic dish decreases. Equation (1.7) shows that the gain of the parabolic dish is related to the square of the diameter; when the diameter is doubled, the gain is four times (6 dB), and when both transmitter and receiver double the size of the dish the total gain will increase by (12 dB).

Figure (1.8) shows a typical radiation pattern of a parabolic reflector antenna with a uniform illumination.
The focal length $f$ of the parabolic dish is calculated from:

$$f = \frac{b^2}{16d} \quad (1.8)$$

Where $d$ is the depth of the parabola at the centre.

The ratio $f/d$ (focal length to diameter of the parabola) is very important in designing the antenna. The value of 0.25 is very common in designing antenna where the focus is in the same plane as the rim of the dish.

2. **Horn Antenna**

The horn antenna is very widely used in microwave applications since the early of 1900s. The name horn antenna comes from the appearance of this type of antennas. The flare of the horn antenna can be square, rectangular, cylindrical, or conical. Horn antennas are very easy fed with waveguide, but it can be fed also by a coaxial cable and a proper transition. Horn antennas are widely used as the active element of the parabolic reflector antenna where the horn is pointed towards the center of the parabolic antenna. The principle of how it works is very simple. If a waveguide is terminated it will radiate energy producing a broad radiated pattern [13] figure (1.9).
By flaring the end of the guide we can get the desired radiation pattern. Flaring can be in the E-plane or H-plane or both, figure (1.10). The dimension of the horn antenna is shown in figure (1.11), the relation between the horn dimensions is:

\[ L = \frac{D^2}{2\lambda} \]  

(1.9)

**Figure (1.10) Horn antennas from [1]**

**Figure (1.11) Horn antenna dimensions**
2. Chapter Two: Microstrip Antennas

2.1. Introduction

The Microstrip Patch Antenna is a single-layer design which consists generally of four parts (patch, ground plane, substrate, and the feeding part). Patch antenna can be classified as single-element resonant antenna. Once the frequency is given, everything (such as radiation pattern input impedance, etc.) is fixed. The patch is a very thin ($t \ll \lambda_0$, where $\lambda_0$ is the free space wavelength) radiating metal strip (or array of strips) located on one side of a thin non-conducting substrate, the ground plane is the same metal located on the other side of the substrate. The metallic patch is normally made of thin copper foil plated with a corrosion resistive metal, such as gold, tin, or nickel. Many shapes of patches are designed some are shown in figure (2.1) and the most popular shape is the rectangular and circular patch. The substrate layer thickness is 0.01–0.05 of free-space wavelength ($\lambda_o$). It is used primarily to provide proper spacing and mechanical support between the patch and its ground plane. It is also often used with high dielectric-constant material to load the patch and reduce its size. The substrate material should be low in insertion loss with a loss tangent of less than 0.005. In this work we have used Arlon AD 410 with dielectric constant of 4.1 and tangent loss of 0.003.

Generally, substrate materials can be separated into three categories according to the dielectric constant $\varepsilon_r$ [1].

1. Having a relative dielectric constant $\varepsilon_r$ in the range of 1.0–2.0. This type of material can be air, polystyrene foam, or dielectric honeycomb.
2. Having $\varepsilon_r$ in the range of 2.0–4.0 with material consisting mostly of fiberglass reinforced Teflon.
3. With an $\varepsilon_r$ between 4 and 10. The material can consist of ceramic, quartz, or alumina.

The advantages of the microstrip antennas are small size, low profile, and lightweight, conformable to planar and non-planar surfaces. It demands a very little volume of the structure when mounting. They are simple and cheap to manufacture using modern printed-circuit technology. However, patch antennas have disadvantages. The main disadvantages of the microstrip antennas are: low efficiency, narrow bandwidth of less than 5%, low RF power due to the small separation between the radiation patch and the ground plane (not suitable for high-power applications).

2.2. Types of Patch Antennas

There are a large number of shapes of microstrip patch antennas; they have been designed to match specific characteristics. Some of the common types are shown in figure (2.1), for millimeter wave frequencies, the most common types are rectangular, square, and circular patches.
Choose of substrate is also important, we have to consider the temperature, humanity, and other environmental ranges of operating. Thickness of the substrate \( h \) has a big effect on the resonant frequency \( f_r \) and bandwidth \( BW \) of the antenna. Bandwidth of the microstrip antenna will increase with increasing of substrate thickness \( h \) but with limits, otherwise the antenna will stop resonating.

### 2.3. Feeding Methods

There are many methods of feeding a microstrip antenna. The most popular methods are:

1. Microstrip Line.
2. Coaxial Probe (coplanar feed).
3. Proximity Coupling.
4. Aperture Coupling.
Because of the antenna is radiating from one side of the substrate, so it is easy to feed it from the other side (the ground plane), or from the side of the element.

The most important thing to be considered is the maximum transfer of power (matching of the feed line with the input impedance of the antenna), this will be discussed later in the section of Impedance Matching.

Many good designs have been discarded because of their bad feeding. The designer can build an antenna with good characteristics and good radiation parameter and high efficiency but when feeding is bad, the total efficiency could be reduced to a low level which makes the whole system to be rejected.

### 2.3.1 Microstrip Line Feed.

This method of feeding is very widely used because it is very simple to design and analyze, and very easy to manufacture. Figure (2.2) shows a patch with microstrip line feed from the side of the patch.

![Figure (2.2) Microstrip patch antenna with feed from side](image)

The position of the feed point ($y_o$) of the patch in figure (2.2b) has been discussed in details in the section of Impedance Matching.

Feeding technique of the patch in figure (2.2a) and figure (2.3) is discussed in [7]. It is widely used in both one patch antenna and multi-patches (array) antennas.

The impedance of the patch is given by [7]:

$$Z_a = 90 \frac{\varepsilon_r^2}{\varepsilon_r - 1} \left( \frac{L}{W} \right)^2 \quad (2.1)$$

The characteristic impedance of the transition section should be:

$$Z_T = \sqrt{50 + Z_a} \quad (2.2)$$

The width of the transition line is calculated from [7]:
The width of the 50Ω microstrip feed can be found using the equation (2.4) below:

\[ Z_o = \frac{120\pi}{\sqrt{\varepsilon_{eff}} \left( 1.393 + \frac{W}{h} + \frac{2}{3} \ln \left( \frac{W}{h} + 1.444 \right) \right)} \]  

(2.4)

Where \( Z_o = 50Ω \)

The length of the strip can be found from (4.24)

\[ R_{in(x=0)} = \cos^2 \left( \frac{\pi}{L} x_o \right) \]  

(2.5)

The length of the transition line is quarter the wavelength:

\[ l = \frac{\lambda}{4} = \frac{\lambda_o}{4\sqrt{\varepsilon_{eff}}} \]  

(2.6)

![Figure (2.3) Rectangular microstrip patch antenna](image)

2.3.2 Coaxial Feed (Coplanar Feed)

Coupling of power to the patch antenna through a probe is very simple, cheap, and effective way. If the designer adjusts the feed point to 50Ω, so he just needs to use a 50Ω coaxial cable with N-type coaxial connector.

The N-coaxial connector is coupled to the back side of the microstrip antenna (the ground plane) and the centre connector of the coaxial will be passed through the substrate and soldered to the patch, as shown in the figure (2.4).
2.3.3 Proximity Coupling

Proximity coupling is use two substrate $\varepsilon_{r1}$ and $\varepsilon_{r2}$. The patch will be on the top, the ground plane in the bottom and a microstrip line is connected to the power source and lying between the two substrates as shown in the figure (2.5). This type is known also as "electromagnetically coupled microstrip feed".

The principle of this mechanism is that the behavior between the patch and the feed strip line is capacitive. Analysis and design of such an antenna is little more complicated than the other
ones discussed in the previous sections because the designer has to take into account the effect of the coupling capacitor between the strip feed line and the patch as well as the equivalent R-L-C resonant circuit representing the patch and the calculating of two substrates ($\varepsilon_{r1}$ and $\varepsilon_{r2}$). The coupling capacitor of this antenna can be designed for impedance matching of the antenna.

### 2.3.4 Aperture Coupling

![Aperture Coupling Diagram](image)

**Figure (2.6)** Aperture coupling feed method

Figure (2.6) shows the layers of the microstrip patch antenna using the aperture mechanism. The ground plane has an aperture in a shape of a circle or rectangular, and separates two substrates: the upper substrate $\varepsilon_{r1}$ with the patch on it, and the lower substrate $\varepsilon_{r2}$ with the microstrip feed line under it. This type of coupling gives wider bandwidth. Another property of this type is the radiating of the feeding strip line is reduced by the shielding effect of the ground plane. This feature improves the polarization purity [8].
3. Chapter Three: Methods of Analysis

There are many methods of microstrip antenna analysis; the most popular are transmission line (in which we assume that the patch is a transmission line or a part of a transmission line).

The second method is the cavity mode (here we assume that the patch is a dielectric – loaded cavity). The transmission line method is the easiest way of studying the microstrip patch antennas. We will discuss briefly each one of these two ways [1].

3.1. Transmission Line Model

![Figure (3.1) Microstrip line](image)

The transmission line method is the easiest way to study the microstrip antenna. In this method the transmission line model represents the microstrip patch antenna by two slots, separated by a low-impedance transmission line of length L. Results we get are not the best accurate compared with other methods but it is good enough to design the antenna.

To study the theory of microstrip transmission line we have two different cases:

W/h < 1 (narrow strip line) and this is not what we are interesting with.

The second case w/h >>1 and \( \varepsilon_r > 1 \) (wider transmission line) this will help us to build a good picture to study the antenna se figure (3.1).
The first approximation we make is to assume that the thickness of the conductor $t$ that forms the line has no effect on our calculations, because it is very thin comparing with the substrate $h$, ($h >> t$); so we use here empirical formulas that depend only on the line dimensions: The width $W$, the length $L$, the height $h$, and the dielectric constant $\varepsilon_r$ of the substrate [3].

The characteristic impedance of the microstrip line can be written as: [1, 3, 4, 5, 6, and 8]

$$ Z_0 = \frac{120\pi}{\sqrt{\varepsilon_{reff}}} \left( 1.393 + \frac{W}{h} + \frac{2}{3} \ln \left( \frac{W}{h} + 1.444 \right) \right) $$

(3.1)

The width of the microstrip line is given by: [1, 3, 4, 5, 6, and 8]
The microstrip patch antenna in figure (3.2 a and b) looks longer than its physical dimensions because of the effect of fringing. The effective length therefore is differing from the physical length by $\Delta L$. A very popular approximation to calculate the extension of the length of the patch is given by [1, 4, 5, 6, 7, and 8].

$$\Delta L = 0.412 \left( \frac{\varepsilon_{eff} + 0.3}{\varepsilon_{eff} - 0.258} \right) \left( \frac{w}{h} + 0.264 \right) \left( \frac{w}{h} - 0.264 \right)$$  \hspace{1cm} (3.3)

Equation (3.3) shows that the extension of the length $\Delta L$ is a function of the ratio $\frac{w}{h}$ and $\varepsilon_{eff}$.

To calculate the effective length, we add the length $L$ to the extension of the length $\Delta L$.

$$L_{eff} = (L + 2\Delta L_{eff})$$  \hspace{1cm} (3.4)

For $TM_{010}$ the resonant frequency is given by:

$$\left( f_r \right)_{010} = \frac{1}{2L\sqrt{\varepsilon_r/\mu_0 \varepsilon_o}} = \frac{v_o}{2L\sqrt{\varepsilon_r}}$$  \hspace{1cm} (3.4)

Where $v_o$ is speed of light in free space.

It is important to note that the characteristic impedance given by equation (3.1), (3.2), and (3.3) are approximate value.

To estimate the effective dielectric constant let us consider the radiating patch is embedded into the dielectric as shown in the figure (3.3)

**Figure (3.3) Microstrip line embedded into the dielectric**

Assuming the same dimensions of $W$, $h$, and $t$, the effective dielectric constant $\varepsilon_{eff}$ can be defined as: “the dielectric constant of the uniform dielectric material so that the line of
Figure (3.3) has identical electrical characteristics, particularly propagation constant, as the actual line of figure (3.1) [1].

For patch antennas air is above the substrate, this will lead to $1 < \varepsilon_{reff} < \varepsilon_r$. For $\varepsilon_r \gg 1$, $\varepsilon_{reff}$ is closer to the actual value of the dielectric constant $\varepsilon_r$ of the substrate.

The affective dielectric constant is also a function of frequency $f_r$ equation (3.3).

$$f_r = \frac{\nu_0}{2\sqrt{\varepsilon_{reff}} (1 + 2\Delta L_{eff})}$$

(3.5)

Working in high frequencies makes the microstrip line behave more homogeneous line as it is only one dielectric (one substrate under and above the transmission line), and the effective dielectric constant is closer to the actual dielectric constant.

Figure (3.4) shows the variations, as a function of frequency, of the effective dielectric constant of a microstrip line with three different substrates [1].

![Figure (3.4) Effective dielectric constant versus frequency for typical substrate, from [1]](image)

The effective dielectric constant $\varepsilon_{reff}$ can be calculated from the formula: [1, 3, 4, 5, and 6]

$$\varepsilon_{reff} = \frac{\varepsilon_r + 1}{2} + \frac{2\varepsilon_r - 1}{2\sqrt{1 + \frac{12h}{W}}}$$

(3.6)
From the equation (3.6) above we can conclude that the effective dielectric constant is a function of frequency $f_r$, height of the substrate $h$, width of the microstrip $W$ as well as the permittivity of the substrate $\varepsilon_r$.

We can generalize the effective dielectric constant shown in figure (3.4) into the one shown in figure (3.5)

![Effective dielectric constant versus frequency](image)

**Figure (3.5)** Effective dielectric constant versus frequency

$\varepsilon_{eff}$ has to be within the limits of the figure (3.5).

### 3.2. Cavity Model

The cavity model in analyzing the microstrip antennas is based on the assumption that the region between the microstrip patch and ground plane is a resonance cavity bounded by ceiling and floor of electric conductors and magnetic walls along the edge of the conductor as shown in figure (3.6) [1, 8, and 16].

![Magnetic wall model of a microstrip patch antenna](image)

**Figure (3.6)** Magnetic wall model of a microstrip patch antenna
The assumption above is based on the observation of:

1. There are only three field components in the region enclosed by the cavity: $E_z$ component in the $z$ axis, and two components of $H$ along the $x$ and $y$ axis ($H_x$, $H_y$).
2. Because $h$ (height of the substrate) is very thin ($h<<\lambda$), field in the interior region do not vary with $z$-coordinates for all frequencies.
3. The electric current in the microstrip patch has no component normal to the edge of the patch at any point.

This model is fair good in studying the microstrip resonators with the edge extending slightly to account for the fringing field.

Before going further with calculation of the field in the cavity let’s take a look on the mechanism of the cavity.

Consider the microstrip antenna in the figure (3.7) [1].

![Figure (3.7) Charge distribution and current density on a microstrip antenna](image)

When the microstrip antenna is connected to a microwave source, the charge distribution will be established on the upper and the lower planes of the antenna as shown in figure (ca2). The charge distribution is controlled by two mechanisms; attractive and repulsive. The attractive force is between the opposite charges on the patch and on the ground plane, it creates a current density inside the dielectric $J_b$ at the bottom of the patch.

The repulsive force is between the like charges tends to push the charges from the bottom of the patch around the edge of the patch to the top of the patch, this will create the current density $J_t$ as shown in figure (3.7).

In the case of microstrip antennas $W>>h$ the attractive mechanism dominates and at charges concentration will within the dielectric under the patch, and the current flow around the edge
can be neglected because it decreases as the ratio height to width decreases. "this would allow the four side walls to be modeled as perfect magnetic conducting surfaces which ideally would not disturb the magnetic field and in turns the electric field distribution beneath the patch". This good approximation to the cavity model leads us to deal with the side walls as perfect magnetic conducting walls.

We have mentioned before that the field inside the cavity has three field components $E_Z, H_X$ and $H_Y$; the wave equation (3.7) can be re-written as equation (3.8):

$$ \nabla \times \nabla \times \vec{E} - k^2 \vec{E} = -j\omega \mu_0 \vec{J} \quad (3.7) $$

$$ \nabla^2 E_Z + k^2 E_Z = j\omega \mu_0 \vec{z} \cdot \vec{J} \quad (3.8) $$

Where $k^2 = \omega^2 \mu_0 \varepsilon_0 \varepsilon_r$ is the wave number.

$\vec{J}$ = Electric current density fed by the feed line to the patch.

$\vec{z}$ Is the unit vector normal to the plane of the patch.

In addition we have on the top and the bottom conductors:

$$ \hat{n} \times \vec{E} = 0 \quad (3.9) $$

And on the walls:

$$ \hat{n} \times \vec{H} = 0 \quad (3.10) $$

**3.3. The Ground plane**

As a part of the antenna, the ground plane should be infinite in size as for a monopole antenna [7] but in reality this is not easy to apply besides a small size of ground plane is desired.

Length of ground plane should be at least one wavelength, it means as the length of the patch is equal or less than half wavelength ($L \leq \lambda_0/2$) so ground plane will extend $\lambda/4$ from the edge of the patch.

$$ \lambda_0 = \frac{\nu_0}{f_r} \quad (3.11) $$

Where:

$\lambda_0$ is wavelength in free space.

$\nu_0$ is speed of light in free space (3 00000000 m/s).
$f_r$ is resonance frequency (1.57542 GHz).

$$\lambda_{eff} = \frac{\nu_o}{f_r} \sqrt{\varepsilon_{reff}} \tag{3.12}$$

$\lambda_{eff}$ is effective wavelength in the substrate.

$\varepsilon_{reff}$ is effective dielectric constant in the substrate.

$\lambda_{eff} = 96.5857\text{mm}$

$$\lambda_{eff}/4 = 24.146\text{mm} \tag{3.13}$$

The width $W$ of the patch must be less than the wavelength in the dielectric substrate material so that higher – order modes will not be excited.

An exception to this condition, multiple feeds are used to eliminate higher – order modes, this is not discussed in this work [2].
4. Chapter Four: Antenna Parameters

4.1. Radiation Pattern

The radiation pattern is defined as “a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates” [1].

![Figure (4.1) Coordinate system for antenna analysis, from [1]](image)

\[ P_{rad} = \frac{1}{2} \text{Re} \iint_{s} E \times H^* \cdot ds \]  \hspace{1cm} (4.1)

\[ = \frac{1}{2\eta} \iint (|E_{\theta}|^2 \times |E_{\theta}^*|^2) r^2 \sin \theta \, d\theta \, d\phi \]  \hspace{1cm} (4.2)

We will not consume too much time to prove the equations (4.1) (4.2) above, they have been discussed in details in the references of this report, we want to mention now the fact that for a microstrip antenna, the electric field \( E \) within the patch is normal to the patch and the ground plane, and the magnetic field \( H \) is parallel to the strip edge. Polarization of a rectangular patch antenna for the dominant mode is linear and directed along the patch dimensions [1, 4, and 8].

4.2. Efficiency and Quality Factor

For a microstrip patch antenna, efficiency can be defined as the power radiated from the microstrip element divided by the power received by the input to the element. Factors that affect the efficiency of the antenna and make it high or low are the dielectric loss, the conductor loss, the reflected power (Voltage Standing Wave Ratio VSWR), the cross-polarized loss, and power dissipated in any loads in the element.
General expression of the radiation efficiency can be found in most books of antennas including references of this research [1, 3, 4, 5, 6, and 8]:

\[ e = \frac{P_{rad}}{P_{rec}} \]  \hspace{1cm} (4.3)

Where:

- \( P_{rad} \) = Power radiated by the antenna.
- \( P_{rec} \) = Power accepted by the antenna.

Efficiency can also be expressed in terms of the quality factor \( Q \) as follows: [1, 4, and 8]

\[ e = \frac{1}{Q_{rad}} = \frac{Q_t}{Q_{rad}} \]  \hspace{1cm} (4.4)

\( Q_t \) = total quality factor

\( Q_{rad} \) = quality factor due to radiation (space wave) losses.

\[ \frac{1}{Q_t} = \frac{1}{Q_{rad}} + \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_{sw}} \]  \hspace{1cm} (4.5)

\( Q_c \) = quality factor due to conduction losses (ohmic).

\( Q_d \) = quality factor due to dielectric losses.

\( Q_{sw} \) = quality factor due to surface waves.

Most microstrip antennas have efficiency of between 80 to 90 percent. For a very thin antenna \( h \ll \lambda_o \), \( t \ll \lambda_o \).

There are approximate formulas to calculate the quality factor [1]:

\[ Q_c = h\sqrt{\pi f\sigma \mu} \]  \hspace{1cm} (4.6)

\[ Q_d = \frac{1}{\tan \delta} \]  \hspace{1cm} (4.7)

\[ Q_{rad} = \frac{2\omega \varepsilon_r}{hGt/l} K \]  \hspace{1cm} (4.8)

Where:

- \( \tan \delta \) = the loss tangent of the substrate.
- \( \sigma \) = conductivity of conductors.
- \( Gt/l \) = the total conductance per unit length.
For a rectangular aperture operating in the dominant TM010 mode:

\[ K = \frac{L}{4} \quad (4.9) \]

\[ \frac{Gt}{l} = \frac{G_{rad}}{W} \quad (4.10) \]

### 4.3. Directivity and Gain

Directivity is the ability of an antenna to focus energy in a particular direction. The definition of the directivity according to IEEE Standard 145-1983: “Directivity (of an antenna) (in a given direction) is the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions”. Note that the radiation intensity is equal to the total power radiated by the antenna divided by \(4\pi\). Directivity is always greater than one [8].

\[ D = \frac{\frac{1}{2}Re(E_\theta H'_\varphi - E_\varphi H'_\theta)(\theta = 0)}{P_{rad}/4\pi} \quad (4.11) \]

Where:

\[ P_{rad} = \text{Radiated power.} \]

\[ \eta_o = 120\pi \quad (\Omega) \]

A good approximation of equation (4.11) for the directivity \(D\) of a rectangular patch antenna is given by [8, and 1]. Note that \((\theta = 0)\) in this approximation.

\[ D \approx \frac{4(k\omega W)^2}{\pi \eta G_{rad}} \quad (4.12) \]

Where \(G_{rad} = \text{radiation conductance of the patch [1, 4, 5, and 9].}\)

The directive gain (according to IEEE Std 145-1983) is “the ratio of the radiation intensity, in a given direction to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically”. We can obtain gain from directivity of the antenna:

\[ G = eD \quad (4.13) \]

\(e\) is the efficiency of the antenna. Gain is always less than directivity because efficiency is between 0 and 1. The directivity increases with increase in substrate thickness \(h\) and patch width \(W\). Conversely the beamwidth is expected to decrease with increasing of \(h & W\) [8].
4.4. Impedance Matching

The theory of maximum power transfer states that for the transfer of maximum power from a source with fixed internal impedance to the load, the impedance of the load must be the same of the source. “Jacobi’s low”

\[ Z_S = Z_L^* \quad (4.14) \]

\( Z_S \) = impedance of the source.

\( Z_L \) = impedance of the load.

(*) indicates the complex conjugate.

Most microwave applications are designed with an input impedance of 50Ω, so matching the antenna to 50Ω is our desire.

We can begin with representing the patch by a parallel equivalent admittance \( Y \) as shown in figure (4.2).

![Figure (4.2) Rectangular patch and its transmission model equivalent](image)

Where:

\[ Y = \frac{1}{Z_L} = G + jB \quad (4.15) \]

Here we have [1]

\[ Y1 = Y2 \quad (4.16) \]

That’s mean:
\[ G_1 = G_2, \quad B_1 = B_2 \quad \text{(4.17)} \]

A general expression for the conductance \( G_1 \) is given by [1, 4, 5, and 8]

\[ G_1 = \frac{2P_{rad}}{|V_o|^2} \quad \text{(4.18)} \]

Where \( P_{rad} \) is the radiation power (defined before).

\( V_o \) is the voltage across the slot.

\[ P_{rad} = \frac{|V_o|^2}{2\pi\eta_o} \int_0^\pi \left( \sin\left(\frac{k_0 W}{2}\cos\theta\right) \right)^2 \sin^3 \theta \, d\theta \quad \text{(4.19)} \]

Therefore \( G_1 \) can be expressed as:

\[ G_1 = \frac{1}{120\pi} \int_0^\pi \left( \sin\left(\frac{k_0 W}{2}\cos\theta\right) \right)^2 \sin^3 \theta \, d\theta \quad \text{(4.20)} \]

Going back to equation (4.14):

\[ Z_s = R_s + jX_s = Z'_L = R_L - jX_L \quad \text{(4.21)} \]

\[ Z_s = \frac{1}{Y_s} = R_s = \frac{1}{2g_1} \quad \text{(4.22)} \]

According to C. Balanis [1] taking into account the mutual effects of the parallel equivalent admittance \( Y_1 \) and \( Y_2 \) shown in figure (4.2):

\[ R_{in} = \frac{1}{2(G_1 + G_2)} \quad \text{(4.23)} \]

Where “the (+) sign is used for modes with odd (antisymmetric) resonant voltage distribution beneath the patch and between the slots while the minus (-) sign is used for modes with even (symmetric) resonant voltage distribution” [1].

\[ G_{12} = \frac{1}{|V_o|^2} \iint_S E_1 \times H^{2*} \cdot ds \quad \text{(4.24)} \]

Where \( E_1 \) is the electric field radiated by \( Y_1 \)

\( H^{2*} \) is the magnetic field radiated by \( Y_2 \)

\[ G_{12} = \frac{1}{120\pi} \int_0^\pi \left( \sin\left(\frac{k_0 W}{2}\cos\theta\right) \right)^2 J_0(k_0 L \sin \theta) \sin^3 \theta \, d\theta \quad \text{(4.25)} \]

\( J_0 \) is the Bessel function of the first kind of order zero.
Finally the position of feed point of the patch (where the impedance of the patch at that point is 50\(\Omega\)) can be found from the following equation:

\[
Rin = \frac{1}{2(G1+G12)} \cos^2\left(\frac{\pi}{L} y_0\right)
\]  
(4.26)

![Figure (4.3)](image)

Figure (4.3) Dimensions of the patch

From equations (4.18) ~ (4.26) we can note that the impedance of the microstrip patch antenna is not depending on the substrate dielectric constant \(\varepsilon_r\) or the height of it \(h\). The resonance input resistance is depending strongly on the width \(W\) of the patch, increasing \(W\) will decrease the input resistance of the patch \(Rin\) figure (4.3).

### 4.5. Return Loss

Return loss is an important parameter when testing an antenna. It is related to impedance matching and the maximum transfer of power theory. It is also a measure of the effectiveness of an antenna to deliver of power from source to antenna. The return loss (RL) is defined by the ratio of the incident power of the antenna \(P_{in}\) to the power reflected back from the antenna of the source \(P_{ref}\); the mathematical expression is:

\[
RL = 10 \log_{10} \frac{P_{in}}{P_{ref}} (dB)
\]  
(4.27)

For good power transfer, the ratio \(\frac{P_{in}}{P_{ref}}\) is high. Another definition of return loss we can get from equation (4.27) is the difference in dB between the power sent towards the antenna and
the power reflected from it. It is always positive when the antenna is passive and negative when it is active.

We can find the equation (5.27) written in terms of voltage and voltage-standing-wave-ratio (VSWR) and impedance as follows [18]:

\[
RL = 10 \log_{10} \left| \frac{1}{\rho} \right| \quad (dB) \quad (4.28)
\]

\[
= -20 \log_{10} |\rho| \quad (dB) \quad (4.29)
\]

\[
RL = 20 \log_{10} \left| \frac{V_{SWR} + 1}{V_{SWR} - 1} \right| \quad (dB) \quad (4.30)
\]

\[
= (40 \log_{10} e) \text{artanh} \left( \frac{1}{V_{SWR}} \right) \quad (dB) \quad (4.31)
\]

\[
RL = 20 \log_{10} \left| \frac{Z_1 + Z_2}{Z_1 - Z_2} \right| \quad (dB) \quad (4.32)
\]

Where \( \rho \) is the complex reflection coefficient at the input of the antenna.

VSWR is the voltage standing wave ratio.

\( Z_1 \) and \( Z_2 \) is the impedance of the source and the antenna.

4.6. **Polarization**

The polarization of an antenna is the polarization of the wave radiated from the antenna. A receiving antenna has to be in the same polarization as the transmitting antenna otherwise it will not resonate. Polarization is a property of the electromagnetic wave; it describes the magnitude and direction of the electric field vector as a function of time, with other words “the orientation of the electric field for a given position in space”. A simple strait wire has one polarization when mounted vertically, and different polarization when mounted horizontally figure (4.4).

Polarization can be classified as linear, circular, and elliptical. In linear polarization the antenna radiates power in the plane of propagation, only one plane, the antenna is vertically linear polarized when the electric field is perpendicular to the earth’s surface, and horizontally linear polarized when the electric field is parallel to the earth’s surface.

Circular polarization antenna radiates power in all planes in the direction of propagation (vertical, horizontal, and between them). The plane of propagation rotates in circle making one complete cycle in one period of wave.

From Maxwell’s Equations:

\[
\nabla \times E = -j\omega \mu H \quad (4.33)
\]

\[
\nabla \times H = (\sigma + j\omega \epsilon)E \quad (4.34)
\]
\[ \nabla \cdot E = \frac{\rho}{\varepsilon} \quad (4.35) \]
\[ \nabla \cdot H = 0 \quad (4.36) \]

A well known solution of equation (4.33) is as follows [1, 3, 4, 6, 7, and 10]:
\[ \nabla^2 E - \gamma^2 E = \nabla (\rho / \varepsilon) \quad (4.37) \]

In free space (\( \rho = 0 \))
\[ \nabla^2 E - \gamma^2 E = 0 \quad (4.38) \]

Equation (4.38) is known as Wave Equation, and there are many possible solutions to it [7]; one possible solution is:
\[ E = E_0 e^{j \omega t + \gamma z} \hat{\lambda} \quad (4.39) \]
\[ E = E_0 e^{j \omega t + \gamma z} \hat{\gamma} \quad (4.40) \]
\[ E = E_0 e^{j \omega t \pm \gamma x} \hat{\lambda} \quad (4.41) \]

Now consider the figure (4.4); the electromagnetic wave radiated by an antenna has an electric field \( E \) with two components \( E_x \) and \( E_y \), where:
\[ E_x = |E_x| \cos (\omega t - \beta z) \quad (4.42) \]
\[ E_y = |E_y| \cos (\omega t - \beta z + \varphi) \quad (4.43) \]

**Figure (4.4)** Polarization of electromagnetic wave
$|E_x|$ and $|E_y|$ are the amplitude of the field components in the directions of $x$ and $y$ respectively. Equations (4.42) and (4.43) are another possible solution of the wave equation (4.38). The components $|E_x|$ and $|E_y|$ of equations (4.42) and (4.43) describes the type of polarization of the electromagnetic wave and the antenna; when $|E_{yx}|$ or $|E_{y}|$ is zero, the wave and antenna are said to be linear polarized; when $|E_x| = |E_y| \neq 0$, the antenna and wave are circularly polarized; when $|E_x| \neq |E_y| \neq 0$ it is elliptically polarized. Linear polarization is used in application like TV sending. Circular polarization is widely used in satellite communication because linear polarization is poor and hard to match in satellite transmission because of what called Faraday Rotation to the electromagnetic wave, which means “linear polarized electromagnetic wave may be rotated by an unknown amount (depend on the thickness and temperature of the ionosphere, as well as the frequency; the rotation is high at lower frequencies and small at higher frequencies)”. 
5. Chapter Five: Results and Discussion

5.1. Design Methods

After the discussion of the simplified formulation in previous sections, the procedure for designing a rectangular microstrip patch antenna is explained. In this procedure there are three essential parameters for the design: the frequency of operation \( f \), the dielectric constant of the substrate \( \varepsilon_r \) and the height of the dielectric substrate \( h \).

For a given \( \varepsilon_r \) and \( h \), we design a rectangular microstrip antenna for the resonant frequency \( f_r \) (finding the width and length of the patch).

A. Microstrip patch antenna of figure (2.2 b).

1. From equation (3.2) we calculate \( W \):

\[
W = \frac{1}{2fr\sqrt{\mu_0\varepsilon_0}} \sqrt{\frac{2}{\varepsilon_r+1}} = \frac{v_0}{2fr} \sqrt{\frac{2}{\varepsilon_r+1}}
\]

(5.1)

Now determine the effective constant of the microstrip antenna from equation (3.6):

\[
\varepsilon_{\text{eff}} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2\sqrt{1+\frac{12h}{W}}}
\]

(5.2)

Use equation (3.3) to determine the extension of \( \Delta L \):

\[
\Delta L_{\text{eff}} = 0.412 \frac{(\varepsilon_{\text{eff}}+0.3)(\frac{W}{h}+0.264)}{(\varepsilon_{\text{eff}}-0.258)(\frac{W}{h}+0.8)}
\]

(5.3)

The actual length of the patch can be found from (3.5):

\[
L = \frac{1}{2fr\sqrt{\varepsilon_{\text{eff}}\mu_0\varepsilon_0}} - 2\Delta L
\]

(5.4)

2. Calculation of the feed point

We want to match the antenna to 50Ω; \( R_{in} = 50Ω \).

Use equations (4.21), (4.24), (4.18), (4.23) discussed earlier to determine the feeding point location.

\[
R_{in} = \frac{1}{2(G1+G12)}
\]

(5.5)

\[
R_{in} = \frac{1}{2(G1+G12)} \cos^2\left(\frac{\pi}{L}y_0\right)
\]

(5.6)

\[
G1 = \frac{L1}{120\pi^2}
\]

(5.7)
\[ I_1 = \int \left[ \sin \left( \frac{k_0 W}{2} \cos \theta \right) \right]^2 \sin^3 \theta d \theta \]  

(5.8)

\[ G_{12} = \frac{1}{120n^2} \int \left[ \sin \left( \frac{k_0 W}{2} \cos \theta \right) \right]^2 J_0(k_0 L \sin \theta) \sin^3 \theta d \theta \]  

(5.9)

3. Calculating the dimensions of the ground plane.

\[ \lambda_{eff} = \frac{v_0}{f_r} \sqrt{\epsilon_{reff}} \]  

(5.10)

Length of ground plane \( \geq \left( \frac{\lambda_{eff}}{4} \right) \times 2 + L \)

Width of ground plane \( \geq \left( \frac{\lambda_{eff}}{4} \right) \times 2 + W \)

**B. Microstrip patch antenna of figure (2.2) a.**

1. Calculating W and L from equations (5.1), (5.2), (5.3), (5.4)

2. The feeding method is now via a microstrip transition line.

Find the characteristic impedance of the patch from:

\[ Z_a = 90 \frac{\epsilon_r^2}{\epsilon_r - 1} \left( \frac{L}{W} \right)^2 \]  

(5.11)

Impedance and transition section:

\[ Z_T = \sqrt{50 + Z_a} \]  

(5.12)

Width of transition line \( W_T \):

\[ Z_T = \frac{60}{\sqrt{\epsilon_r}} \ln \left( \frac{8d + W_T}{W} \right) \]  

(5.13)

Length of transition line:

\[ l = \frac{\lambda}{4} = \frac{\lambda_o}{4 \sqrt{\epsilon_{reff}}} \]  

(5.14)

Width of 50Ω microstrip transmission line:

\[ Z_o = \frac{120 \pi}{\sqrt{\epsilon_{reff} \left( 1.393 + \frac{W}{h} + \frac{2}{3} \ln \left( \frac{W}{h} + 1.444 \right) \right)}} \]  

(5.15)

Where \( Z_o = 50 \Omega \)

Length of the microstrip transmission line

\[ R_{in(x=0)} = z_o \frac{z_o}{Z_T} = \cos^2 \left( \frac{\pi}{L} x_o \right) \]  

(5.16)
Two Matlab codes were written according to the two procedures described above can be found in Appendix A and B.

5.2. Example 1 GPS Receiver Patch Antenna

The GPS system (Global Positioning System) uses 24 satellites called for space vehicles (SV) [10 and 12]. The GPS receiver calculates the position in real time using the signal sent by SV satellite. The message coming from the satellite contains time, distance from satellite and the general system health of all satellites.

The 24 satellites are placed in six orbital planes, four satellites in each, equally spaced and included at a 55 degree relative to the equal orbital plane of the earth. Each satellite repeats its position in the orbit twice every 24 hours. The GPS system provides the user to be visible to five to eight satellites from any point on the earth at every time.

The altitude of the orbiting is approximately 20,200 kilometers from earth surface.

There are many methods of calculating the position; every GPS manufacturer has an own algorithm. A basic concept to understand the location finding is as follows:

To calculate the position we need to be seen to at least three satellites.

Consider that the GPS receiver is within the region of three satellites, each satellite is at the centre of a circle and the GPS receiver is on the surface of this circle, as shown in figure (5.1).

![Figure (5.1) position of GPS receiver](image)
The position \( p \) is a function of the time (or bias) \( b \), clock’s uncorrected time \( c \), and distance to the satellite \( x, y, z \).

\[
p_{i} = \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2 - bc}
\]  

(5.17)

\( i = 1,2,3,4 \)

We will not go further with position calculation because it is not our big matter. What we want from the above discussion is to define GPS system and to mention that we need at least three satellites and to say there are five to eight satellites in sight. This helps us to understand why we can use linear polarized rectangular antenna in GPS receiver. Because of there will be at least five satellites (at normal there are eight to ten satellites) so the GPS receiver is receiving signal from all direction. There is no problem in receiving GPS signal from satellite because GPS systems have employed circular polarization (not linear polarization) for transmission.

To design a rectangular microstrip patch antenna we decide the substrate material and the thickness of it. Frequency of GPS signal is 1575.42 MHz

Set these parameters as follows:

\( \varepsilon_r = 4.1, h = 1.6\, mm, f = 1.57542 \, GHz \)

Apply equations (5.1) to (5.10) to find out patch dimensions:

\( W = 59.716\, mm \)

\( L = 46.798\, mm \)

\( y_o = 17.24\, mm \)

\( \varepsilon_r = 4.1 \)

\( \lambda_{eff} = 0.1157\, mm \)

Equations (5.2) (5.10):

\[
\frac{\lambda_{eff}}{4} = a = 24.146\, mm
\]

Figure (5.2) shows the antenna with its dimensions.
Figure (5.2) The GPS microstrip antenna with its physical dimensions

The thickness of the metal (patch) is negligible because it has not big effect on calculations and analysis, besides it is not seen in our equations (5.1) to (5.16).

A Matlab code was written to simplify the calculation (appendix A).

An antenna with the above dimensions is designed using Sonnet Software 12.56.

The parameters of the antenna have not come as desired because all equations (5.1) to (5.16) are approximate equations (as we mentioned before) [1, 4, 5, and 8], we had to adjust the values of \( W \), \( L \), and \( y_o \) to get best results. After several trials, we get:

\[
W = 57\text{mm} \\
L = 46\text{mm} \\
y_o = 15.5\text{mm}
\]

Box which contains the antenna is made too large (400mm) in side length and 20mm in height to minimize the effect of the side walls on the patch.

Sonnet software is a very good tool to study the antenna but it is slow and needs a very powerful computer when running (we used a computer with X5460 processor of 3.16 GHz and 16 GB RAM memory). It took several hours to design the antenna with high accuracy.

The response of the antenna is shown in figure (5.3)
Figure (5.3) frequency response of the GPS patch antenna

Figure (5.3) shows the response of the patch antenna at resonant frequency of 1.5742 GHz. The GPS frequency 1.57542 GHz is in the region of -21.5026 dB. The input impedance of the antenna is 48.24701 Ω at the resonant frequency (1.57542 GHz) where the 50 Ω impedance is at f = 1.56495 GHz and f = 1.57465 GHz as shown in figure (5.4).

VSWR = 1.183678, loss factor = 0.007075 (magnitude) or -21.506 dB. See figure (5.4)

Figure (5.5) shows the current distribution at different frequencies.
Figure (5.4) input impedance of GPS patch antenna

Current density distribution is shown in figure (5.5)

Figure (5.5) Current density on the patch at different frequencies
The far field pattern can be seen in figures (5.6) to (5.10) below. Theta and phi of the pattern are plotted in polar, Cartesian, and surface coordinates.

**Figure (5.6)** theta in cartesian coordinates

**Figure (5.7)** theta in polar
**Figure (5.8)** phi in cartesian

**Figure (5.9)** phi in polar

**Figure (5.10)** surface graph of theta and phi
<table>
<thead>
<tr>
<th>X</th>
<th>RL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-3.88531</td>
</tr>
<tr>
<td>1</td>
<td>-3.91705</td>
</tr>
<tr>
<td>2</td>
<td>-3.99260</td>
</tr>
<tr>
<td>3</td>
<td>-4.11162</td>
</tr>
<tr>
<td>4</td>
<td>-4.27719</td>
</tr>
<tr>
<td>5</td>
<td>-4.48937</td>
</tr>
<tr>
<td>6</td>
<td>-4.77335</td>
</tr>
<tr>
<td>7</td>
<td>-5.14247</td>
</tr>
<tr>
<td>8</td>
<td>-5.66119</td>
</tr>
<tr>
<td>9</td>
<td>-6.25035</td>
</tr>
<tr>
<td>10</td>
<td>-6.95342</td>
</tr>
<tr>
<td>11</td>
<td>-7.99753</td>
</tr>
<tr>
<td>12</td>
<td>-9.42002</td>
</tr>
<tr>
<td>13</td>
<td>-11.78816</td>
</tr>
<tr>
<td>13.5</td>
<td>-13.1739</td>
</tr>
<tr>
<td>14</td>
<td>-15.0533</td>
</tr>
<tr>
<td>14.5</td>
<td>-17.5222</td>
</tr>
<tr>
<td>15</td>
<td>-20.4145</td>
</tr>
<tr>
<td>15.5</td>
<td>-21.5026</td>
</tr>
</tbody>
</table>

**Table (5.1)** return loss versus position of feed point.

**Figure (5.11)** the effect of changing the position of feed point on return loss.
Figure (5.12) different values of return loss according to feed point position
The table (5.1) and figure (5.11) and (5.12) show the effect of the position of the feed on the return loss of the patch antenna. In sonnet we have kept the y position of the feed point fixed in the middle of the width W and changed the position of the feed point along the x axis from 0 to the point we get maximum value of power transfer at 15.5 mm.

Figure (5.10) shows that the return loss increases as we approach the position of 50Ω input impedance. Figure (5.11) shows different values of return loss at different positions of feed point.

The antenna of this example with its dimensions: \( W = 57\text{mm}, \quad L = 46\text{mm}, \quad y_o = 15.5\text{mm}, \quad \varepsilon_r = 4.1 \) has been manufactured by COGRA, www.cogra.se, the antenna can be seen in figure (5.17).

The test of the antenna has been done in the Technical Research Institute of Sweden (SP), www.sp.se. The test is to couple the antenna to a spectrum analyzer (Vector Network Analyzer Rohde & Schwarz ZVRE) and measure the S11 parameters (scatterings parameters).

**Scattering Parameters**

Scattering parameters or S-parameters (the elements of a scattering matrix or S-matrix) describe the electrical behavior of linear electrical networks when undergoing various steady state stimuli by electrical signals [23].

\[
\begin{pmatrix}
    b_1 \\
    b_2
\end{pmatrix} =
\begin{pmatrix}
    S_{11} & S_{12} \\
    S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
    a_1 \\
    a_2
\end{pmatrix}
\]  

(5.18)

Where:

\[
\begin{align*}
    b_1 &= S_{11} a_1 + S_{12} a_2 \\
    b_2 &= S_{21} a_1 + S_{22} a_2
\end{align*}
\]  

(5.19) \hspace{1cm} (5.20)

Where:
\( S_{11} \) is the input port voltage reflection coefficient.

\( S_{12} \) is the reverse voltage gain.

\( S_{21} \) is the forward voltage gain.

\( S_{22} \) is the output port voltage reflection coefficient.

Input return loss \( RL_{in} \) is a scalar measure of how close the actual input impedance of the network to the nominal system impedance value and, expressed in logarithmic magnitude, is given by

\[
RL_{in} = 20 \log_{10} |S_{11}| \text{ dB} \tag{5.21}
\]

Recall other definitions of return loss in equations: (4.25) to (4.30).

We can use the sonnet software to see the S11 parameters on graph:

Figure (5.13) return loss of the measured antenna.
Figure (5.14) input impedance of the measured antenna

Figure (5.15) voltage standing wave ratio VSWR of the measured antenna
Figure (5.16) frequency response of the measured antenna on Smith chart

Figure (5.17) shows the actual antenna

Figure (5.17) the measured antenna
If we want to see the effect of changing the dielectric constant on the dimensions of the antenna we can keep the thickness of the substrate at 1.6 mm and change the dielectric from 1 to 20 using equations (5.1) to (5.10) as follows:

<table>
<thead>
<tr>
<th>$\varepsilon_r$</th>
<th>d mm</th>
<th>W mm</th>
<th>L mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.6</td>
<td>95.2</td>
<td>92.9</td>
</tr>
<tr>
<td>2</td>
<td>1.6</td>
<td>77.7</td>
<td>66.4</td>
</tr>
<tr>
<td>4.1</td>
<td>1.6</td>
<td>57</td>
<td>46</td>
</tr>
<tr>
<td>8</td>
<td>1.6</td>
<td>44.8</td>
<td>33.5</td>
</tr>
<tr>
<td>10</td>
<td>1.6</td>
<td>40.6</td>
<td>30</td>
</tr>
<tr>
<td>12</td>
<td>1.6</td>
<td>37.3</td>
<td>27.4</td>
</tr>
<tr>
<td>16</td>
<td>1.6</td>
<td>32.6</td>
<td>23.7</td>
</tr>
<tr>
<td>20</td>
<td>1.6</td>
<td>29.4</td>
<td>21.2</td>
</tr>
</tbody>
</table>

We can see that changing the dielectric constant will change the dimensions of the antenna strongly while the change of the thickness will change the length of the antenna slightly but no change on the width, see equations (3.2), (3.5), and (3.6).

We can also redesign the microstrip patch antenna in example 1 using substrate with dielectric constant of 16, the dimensions are:

$W = 31\, \text{mm}$

$L = 22.8\, \text{mm}$

$y_0 = 8.7\, \text{mm}$

The frequency response is as follows:

**Figure (5.18)** frequency response of the patch antenna of dielectric constant of 16
The input impedance versus frequency is as shown in figure (5.19) below:

Figure (5.19) the input impedance of the patch antenna

The radiation pattern can be seen in figures (5.20) to (5.24) below:

Figure (5.20) gain-theta cartesian
Figure (5.21) gain-theta polar

Figure (5.22) gain-phi cartesian

Figure (5.23) gain-phi polar
The center frequency of this microstrip antenna is 1.5754 GHz; the input impedance of the resonance frequency (1.57542 GHz) is 54.5Ω, the frequencies of the 50 Ω impedance are 1.5725 GHz and 1.5765 GHz.

5.3. Example 2: Satellite TV Signal

We take now another example with satellite communications. A TV signal from satellite.

We have defined the SV system (space vehicles) as a system consists of 24 satellites at a height of 20200 kilometres from earth surface.

The signal we are studying now is a television signal coming from the satellite Badr at 26 degree east. Badr is classified as Geostationary Satellite.

“According to Kepler’s law, the orbital period of a satellite varies as the radius of the orbit to the 3/2 power”, the higher the orbit, the longer the period [14]. The satellite which is close to earth surface it needs 90 minutes to complete a cycle around the earth. For 35800 kilometres, it needs 24 hours, at a height of 38400 kilometres the orbital period is then a month (distance between earth and moon is varying between 363,104 km and 405,696 km) [15].

It was proved earlier that satellites at an altitude of 35800 km from earth surface moving in a circular equational orbit can be seen as motionless in sky, no tracking is needed; this type of satellites is called geostationary earth orbits satellites GEO.
Badr satellites are owned and operated by Pakistan and Saudi Arabia and is weighing up to 4000 kg and consuming several kilowatts of electric power produced by solar panels. The fuel of the motors exhausted in 10 years.

Modern satellites have around 40 transponders with 80 MHz bandwidth each. The satellites Badr 4/5/6 sends television channels in frequencies from 10.73 GHz to 12.74 GHz. In our example we will focus on 11,919 GHz signal, which is the group channels mbc (se www.mbc.net), and 11,843 GHz which is the group channels Rotana (www.rotana.net), as well as the rest of the channels in the frequency band of the Badr satellite.

In a very common low cost micro station used in homes there is an antenna with about 1 meter diameter and 1 watt of power.

**Calculations**

We want to receive the frequency 11.862 GHz of Rotana satellite channel. We use equations (2.1) to (2.6) and (5.1) to (5.10) to get the antenna specifications:

Using our MatLab’s code in Appendix B:

Set frequency to 12 GHz, height of substrate 1.6 mm, permeability $\varepsilon_r$ 4.1

We get the following results:

$W = 7.8278$ mm

$L = 5.3875$ mm

The width of the 50 ohms microstrip feed line $= 1.4068$ mm

The length the 50 ohms microstrip feed line $= 2.5456$ mm

We round these values to:

$W = 7.9$ mm

$L = 5.3$ mm

The width of the 50 ohms microstrip feed line $= 1.4$ mm

The length the 50 ohms microstrip feed line $= 2.5$ mm

Note that we rounded the width $W$ to 7.9 with one digit after the comma because we have to take to account the accuracy of manufacturing of the patch, dimensions with more than one digit after the comma are not easy to get in the factory. The condition of the length $L$ to be equal to or less than half the effective wavelength is satisfied here.

If we use the equations (5.2) to (5.5):
\[
\begin{align*}
\varepsilon_{\text{eff}} &= 3.33876 \\
\lambda_{\text{eff}} &= 13.74 \text{ mm} \\
\Delta L &\leq 5.4619 \text{ mm}
\end{align*}
\]

The response of the antenna is as shown in figure (5.25).

![Figure (5.25) frequency response of a patch antenna](image)

Note that the resonant frequency in the graph of figure (5.25) above is not the required 12 GHz. We can explain this with the fact that the calculations are not accurate because the relations 1 to 13 are approximate relations as we mentioned before in chapter 3, but it shows good results.

From equations (5.1) and (5.4) one can realize that the resonant frequency of the patch antenna has inverse relation with length and width of the patch.

\[
W = \frac{1}{2fr\sqrt{\mu_0\varepsilon_0}} \sqrt{\frac{2}{\varepsilon_r+1}} = \frac{v_0}{2fr} \sqrt{\frac{2}{\varepsilon_r+1}} 
\]

(5.1)

We know that the effective relative permeability \(\varepsilon_r\) is a function of frequency, equation (5.4).
If we assume that the effect of change of frequency on the effective relative permeability much less than effect of change of frequency on width and length so we can get from equation (5.1) and (5.4)

\[ f \propto \frac{1}{W} \]  \hspace{1cm} (5.18)

\[ f \propto \frac{1}{L} \]  \hspace{1cm} (5.19)

By testing different values of W and L we can get an acceptable response of the microstrip antenna.

**Figure (5. 26)** frequency response of a patch antenna after adjusting the dimensions

The dimensions of this new antenna are \( L = 5.2 \) mm, \( W = 7.7 \) mm, width of the 50 ohms microstrip feed line = 1.3 mm, length the 50 ohms microstrip feed line = 2.6 mm.
We have tested more than 50 times different values for L and W, width and length of strip line, note that the center frequency of the antenna is at 12.02 GHz but we are satisfied because our return loss of the antenna at frequency 11.843 is -15.5473dB. Note also that our other desired frequency of the channel mbc (11.919) has a return loss of -18.748 dB.

Acceptable results have been found as shown from figures (5.26) to (5.31) below, for the far field pattern.

Note that the area of the microstrip patch is \((5.4 \times 5.7 = 31.35 \text{ mm}^2)\), compare with \(1\text{ m}^2\) diameter of a parabolic antenna which can collect up to 1 watt in best conditions. Our microstrip antenna will have 31.35 microwatts.

**Figure (5.27)** far field response (gain versus theta in Cartesian coordinate)
Figure (5.28) far field response (gain versus theta in Polar coordinate)

Figure (5.29) far field response (gain versus phi)
**Figure (5.20)** far field response (gain versus phi in Polar coordinate)

\[ \text{Gain (dBi) - E Total - 12.02 GHz} \]

**Figure (5.31)** surface analysis of theta and phi versus gain
Figure (5.32) current density at different frequencies
6. Chapter Six: Array Antenna

6.1. Introduction

Microstrip antennas are used in arrays as well as single elements [1, 8, and 13]. By using array in communication systems we enhance the performance of the antenna like increasing gain, directivity scanning the beam of an antenna system, and other functions which are difficult to do with the single element.

Feeding of microstrip array antenna is by series-feed network figure (6.1 a), or corporate-feed network figure (6.1 b).

![Figure (6.1) Feed arrangement for microstrip patch array](image)

The figure (6.2) shows the method of using the $\frac{\lambda}{4}$ impedance transformer lines to match the 100Ω patches to a 50Ω transmission line [1].

![Figure (6.2) $\frac{\lambda}{4}$ impedance transformer matching from [1]](image)
6.2. Two Elements Array

Suppose two antenna elements to make an array as in figure (6.1) above. The two elements are fed with current $I_1$ and $I_2$.

$I_1$ and $I_2$ are equal in magnitude but out of phase:

$$I_1 = I_1 \angle \alpha$$ (6.1)

The point of observation is in the far field, the path length difference is $l \cos \alpha$, where $l$ the distance between the two elements is. As it is defined in [1, 9, and 13], the radiation of element 1 at $P$ will lead the radiation of element 2 with angle $\psi$ where:

$$\psi = \beta l \cos \phi + \alpha$$ (6.2)

$\beta$ = phase constant of the transmitted wave.

The total field at $P$ is

$$E = E_1 [1 + \exp(j\psi)]$$ (6.3)

Where $E_1$ is the field at $P$ due to element 1.

The magnitude of the field at $P$ is:

$$|E_\phi| = 2E_1 \cos \left(\frac{1}{2} \psi\right)$$

$$= 2E_1 \cos \frac{1}{2}(\beta l \cos \phi + \alpha)$$
From equation (6.4) we can see that for a given phase difference and a given distance we can change the radiation pattern by changing \( \alpha \).

### 6.3. Linear Array

We have studied a simple array consist of two elements, now if we put more elements in the line of our two elements array, we build a linear array, figure (6.2).

![Figure (6.2) Uniform linear array of n elements](image)

Now consider figure (6.2) of a simple linear array with equal separation between elements \( l \) and equal current in magnitude and equal difference in phase \( I \)

\[
l, l \angle \alpha, l \angle \alpha_1, l \angle \alpha_2, \ldots l \angle \alpha_n
\]

Field at point \( P \) is:

\[
E = E_1[1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \cdots e^{jn\psi}] \quad (6.5)
\]

The magnitude of \( E \) is:

\[
E = E_o \left| \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \right| \quad (6.6)
\]

Where \( \psi = \beta l \cos \phi + \alpha \) \quad (6.7)

The quantity \( \left| \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \right| \) is known as the array factor and it determines the shape of the radiation pattern. The equation (6.7) has a maximum when \( \psi = 0 \) so \( \beta l \cos \phi = -\alpha \).
We can now place the maximum as we wish by choosing $\alpha$ correctly [12]. The phase of each element in this array can be controlled by phase shifter, and the amplitude of the elements is adjusted by an amplifier or attenuator.

### 6.4. Example 3: Microstrip Array Antenna

In example 2 we have made a microstrip patch antenna with the following specifications:

- $W = 7.7$ mm
- $L = 5.2$ mm
- Width of 50$\Omega$ microstrip transmission line = 1.3 mm
- Length of 50$\Omega$ microstrip transmission line = 2.6 mm
- $h = 1.6$ mm
- $\varepsilon_r = 4.1$

We now combine two patches to build a two-element microstrip array antenna as follows:

![Figure (6.3) two elements microstrip array](image)

Consider the figure (6.2) from [1] and use the method of calculating the length and width of transmission line described in chapter 2-equations (2.1 to 2.6) to get the 100$\Omega$ input impedance of each patch in the combination.

Doing the same procedure of analyzing we have done with example 1 and 2; we get the values of $W, L, X_o, l$ mathematically by using the simple code in appendix B and input them in the sonnet, and adjust them every try until we become satisfied with the results.

The figure (6.4) shows the input impedance versus frequency, the figure (6.4 a) shows the input impedance of 100$\Omega$ of a single patch, and figure (6.4 b) shows the input impedance of 50$\Omega$ of the combination of the 2-elements array.
The frequency response of the two-element array is shown in figure (6.4 b). From figure (6.5) we can see that the centre frequency of the antenna has changed from 12 GHz to 11.9 GHz and the return loss of the antenna increased to -41.4 dB which enhances the total performance of the antenna, while the return loss at 12 GHz is -26 dB (in example 2 it is -21 dB). The -10 dB bandwidth is 1.44 GHz begins from 11.22 GHz to 12.65 GHz which is almost the high band frequency of the Badr satellite (HB of Badr is 11.7-12.75 GHz). Looking again at figure (6.5) we can see that all scanned frequencies (from 10 GHz to 13 GHz) are within the region -3 dB.
Figure (6.5) frequency response of two-element array antenna

The graphs of the far field of the two-element array are shown in figures (6.6) to (6.10):

Figure(6.6)
Compare figure (6.8) with figure (5.15) we can see that the far field pattern is not symmetric on its center. As phi changes from 0 to 90 degrees the gain of the antenna changes from -4 to -15, we can see this change more obvious in figure (6.10).

The next task in this example is to make a combination of 4-elements as shown in figure (6.11)

![Four-element microstrip array](image)

**Figure (6.11)** four-element microstrip array

Use equations (2.1) to (2.6) to get the dimensions of 200\(\Omega\) transmission line. Do the same process we done before by using the sonnet; input impedance of 200\(\Omega\) of single element and input impedance of 50\(\Omega\) of 4-elements are shown in figure (6.12), the response of the antenna is in the figure (6.13).
Figure (6.12) input impedance versus frequency of 50 and 200 ohm

Figure (6.13) frequency response of the 4-elements array antenna

The -10 dB bandwidth is 1.38 GHz, the center frequency is 11.98 GHz and at -20.6 dB. The bandwidth of the four-element array is not larger than that for two-element array, not either the gain, but what we get here is the power. Power received by four-elements array is double the power from two-element array, directivity is also better, the pattern in four-element array is narrow and it is good in beam scan applications [1, 8, and 19]. Figures (6.14) to (6.18) show the different properties of the four-element array.
Figure (6.14)

Figure (6.15)

Figure (6.16)
The patch microstrip array antennas of two and four-elements are manufactured and tested; the real dimensions are shown in figure (6.19) and (6.20).

As a conclusion, the aim of this thesis was to design and study a compact microstrip patch antenna for GPS system as well as satellite TV signal and to study the array of a two and four-element array antenna. Changing the dielectric material can change the dimension of the antenna as shown from our first example. The compression between the calculated results and our measurements shows fairly good agreement for the rectangular patch.

Theoretically, the space between the patches has to be more than the half the wavelength to make it (the spacing region) not radiating region. If the spacing is less than the half wavelength so the patches will affect each other and this will need new calculations to the parameters of the antenna like impedance, feed location, pattern…etc.
In our work, we avoided the complex mathematics and used instead tools to test the antenna in different values of spacing between patches to obtain the desired pattern.

**Figure (6.19)** real dimensions of two-element array

**Figure (6.20)** the real dimensions of the four-element array
Appendix A, Matlab code to calculate dimensions of the microstrip patch antenna related to figure (2.2)b

%Program to calculate the parameters to design a rectangular patch antenna
%the user have to feed the values of frequency, dielectric constant, and
%height of the dielectric.
%the program will calculate automatically the width and length of the patch
%and the distance to the feed point

function[] = cal
    global k0 W L
    f = input( 'input frequency f in Ghz:    ');
    Er = input ( 'input dielectric constant of the substrate Er      ');
    h = input( 'input height of substrate h in mm:    ');
    h=h/1000;   %turns height to meters
    f=f*1e9;    % turn frequency to HZ
    c = 3e8;    % speed of light
    k0=2*pi*f/c;    %wave number
    Rin = 50;   %required input impedance of the antenna

    % calculating Width and Length of the Patch
    W = ( c / ( 2 * f ) ) * ( ( 2 / ( Er + 1 ) )^0.5);
    Er_eff = (Er+1)/2 + (( Er -1 )/2)*1/(sqrt(1+(12*(h/W))));
    L_eff = c/(2*f*sqrt(Er_eff));
    a1 = ( Er_eff + 0.3 ) * ( ( W / h ) + 0.264 );
    a2 = ( Er_eff - 0.258 ) * ( ( W / h ) + 0.8 );
    delta_L = (0.412 * ( a1 / a2 )) * h;
    L = L_eff - 2*delta_L;

    % calculating the distance of the inset feed point
    t = 0:pi;
    g1(t);
    I1 = quad(@g1,0,pi);
    G1 = I1/(120*pi*pi);
    g12(t);
    I12 = quad(@g12,0,pi);
    G12 = I12/(120*pi*pi);
    yo = (L/pi)*(acos(sqrt(2*Rin*(G1+G12))));
    str=[char('width = '), num2str(W*1000), ' mm'
    str=[char('length = '), num2str(L*1000), ' mm'
    str=[char('the inset feed point distance = '), num2str(yo*1000), ' mm'
    h=h/100;

%subfunktions
function [f] = g1(t)
    global k0 W
    f = ((sin(k0*W*0.5*cos(t))/cos(t)).^2*(sin(t)).^3);

function [k] = g12(t)
    global k0 W L
    k =((sin(k0*W*0.5*cos(t))/cos(t)).^2)*(sin(t).^3).*BESSELJ(0,k0*L*sin(t));
Appendix B, Matlab code to calculate dimensions of the microstrip patch antenna related to figure (2.2)a

%Program to calculate the parameters to design a rectangular patch antenna
%the user have to feed the values of frequency, dielectric constant, and
%height of the dielectric.
%program will calculate automatically the width and length of the patch
%and the width and length of the transition and transmission feed line.

f = input( 'input frequency f in Ghz:    ');
Er = input ( 'input dielectric constant of the substrate      ');
h = input( 'input height of substrate h in mm:    ');
h=h/1000;
f=f*1e9;    % turn frequency to HZ
c = 3e8;    % speed of light

% calculating Width and Length of the Patch
W = ( c / ( 2 * f ) ) * ( ( 2 / ( Er + 1 ) )^0.5);
Er_eff = (Er+1)/2 + (( Er -1 )/2)*(1/(sqrt(1+(12*(h/W)))))
L_eff = c/(2*f*sqrt(Er_eff));
a1 = ( Er_eff + 0.3 ) * ( ( W / h ) + 0.264 );
a2 = ( Er_eff - 0.258 ) * ( ( W / h ) + 0.8 );
delta_L = (0.412 * ( a1 / a2 )) * h;
L = L_eff - 2*delta_L;
str=['width of the patch = ', num2str(W*1000), ' mm']
str=['length of the patch = ', num2str(L*1000), ' mm']

% Calculating the input impedance of the patch
Zo = 90 * Er^2*(L/W)^2/(Er-1);
% Calculating the strip transition line
Zt=sqrt(50*Zo);
a3=exp(Zt*sqrt(Er)/60); p=-4*h*a3; q=32*h^2;
Wt1=-(p/2) + sqrt((p/2)^2-q);
Wt2=-(p/2) - sqrt((p/2)^2-q); %width of the transition line
Er_t= (Er+1)/2 + (( Er -1 )/2)*(1/(sqrt(1+(12*(h/Wt2)))))
L_t=(c/f)/(4*sqrt(Er_t)); %length of transition line
str=['width of the transition line = ', num2str(Wt2*1000), ' mm']
str=['length of transition line = ', num2str(L_t*1000), ' mm']

% Calculating the 50 ohm transmission line
syms x;
Z0=50;
d=h*1000;
a = 1.393-(120*pi/(Z0*sqrt(Er)));
RR1=inline('x*d+0.667*log(x/d+1.44)+a');
References


[16] Lo, Y.; D. Solomon; and W.F. Richards Theory and experiment on microstrip antennas;


[19] Antenna array systems: electromagnetic and signal processing aspects / by Maria Lanne, Göteborg: Chalmers tekniska högskola, 2005

